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COLLEGE-LEVEL EXAMINATION PROGRAM

Calculus with Elementary Functions

Description of the Examination

The Calculus with Elementary Functions exam covers skills and concepts that are usually taught in a one-year college course in calculus with elementary functions. The major emphasis of the exam is divided equally between topics from differential and integral calculus. Properties of algebraic, trigonometric, exponential, and logarithmic functions as well as limits are also measured. The exam is primarily concerned with an intuitive understanding of calculus and experience with its methods and applications. Knowledge of preparatory mathematics, including algebra, plane and solid geometry, trigonometry, and analytic geometry, is assumed. Students are permitted, but not required, to use a scientific calculator (non-graphing, non-programmable) during the exam.

The exam includes approximately 45 multiple-choice questions to be answered in two separately timed 45-minute sections.

Knowledge and Skills Required

The subject matter of the Calculus with Elementary Functions exam is drawn from the following topics.

	Approximate Percent of Examination
10%	Elementary Functions (algebraic, trigonometric, exponential, and logarithmic)

- Properties of functions
  - Definition, domain, and range
  - Sum, product, quotient, and composition
  - Absolute value, e.g., $|f(x)|$ and $f(|x|)$
  - Inverse
  - Odd and even
  - Periodicity
  - Graphs; symmetry and asymptotes
  - Zeros of a function

Approximate Percent of Examination

- Limits

  Statement of properties, e.g., limit of a constant, sum, product, and quotient

  The number e such that $\lim\limits_{n\to\infty}\left(1+\dfrac{1}{n}\right)^n = e$ and $\lim\limits_{x\to 0}\dfrac{e^x-1}{x}=1$

  Limits that involve infinity, e.g., $\lim\limits_{x\to 0}\dfrac{1}{x^2}$ is nonexistent and $\lim\limits_{x\to\infty}\dfrac{\sin x}{x}=0$

  Continuity

45% Differential Calculus

- The derivative

  Definitions of the derivative; e.g.,

  $$f'(a)=\lim_{x\to a}\frac{f(x)-f(a)}{x-a} \text{ and}$$
  $$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

  Derivatives of elementary functions

  Derivatives of sum, product, quotient (including tan x and cot x)

  Derivative of a composite function (chain rule); e.g., sin (ax + b), ae^{kx}, ln (kx)

  Derivative of an implicitly defined function

  Derivative of the inverse of a function (including Arcsin x and Arctan x)

  Logarithmic differentiation

  Derivatives of higher order

  Statement (without proof) of the Mean Value Theorem; applications and graphical illustrations

  Relation between differentiability and continuity

  Use of L'Hôpital's rule (quotient and indeterminate forms)

Approximate Percent of Examination

- Applications of the derivative

  Slope of a curve; tangent and normal lines to a curve

  Curve sketching: increasing and decreasing functions; relative and absolute maximum and minimum points; concavity; points of inflection

  Extreme value problems

  Velocity and acceleration of a particle moving along a line

  Average and instantaneous rates of change

  Related rates of change

  Newton's method

45% Integral Calculus

- Antiderivatives
- Applications of antiderivatives

  Distance and velocity from acceleration with initial conditions

  Solutions of $y' = ky$ and applications to growth and decay

  Solutions of $f(y)\,dy = g(x)\,dx$ (variables separable)

- Techniques of integration

  Basic integration formulas

  Integration by substitution (use of identities, change of variable)

  Simple integration by parts, such as

  $\int x \cos x \; dx$ and $\int \ln x \; dx$

- The definite integral

  Concept of the definite integral as an area

  Approximations to the definite integral using rectangles or trapezoids

  Definition of the definite integral as the limit of a sum

  Properties of the definite integral

  The fundamental theorem —

  $$\left(\frac{d}{dx}\int_a^x f(t)\,dt = f(x) \text{ and } \int_a^b f(x)\,dx = F(b) - F(a), \text{ where } F'(x) = f(x)\right)$$

Approximate Percent of Examination

- Applications of the integral
  - Average value of a function on an interval
  - Area between curves
  - Volume of a solid of revolution
    - (disc, washer, and shell methods) about the x- and y-axes or lines parallel to the axes

Sample Questions

The following 25 questions are provided to give an indication of the types of questions that appear on the Calculus with Elementary Functions exam. CLEP exams are designed so that average students completing a course in the subject can usually answer about half the questions correctly.

Before attempting to answer the sample questions, read all the information about the Calculus with Elementary Functions exam on the preceding pages. Additional suggestions for preparing for CLEP exams are provided in Chapter 1.

Try to answer correctly as many questions as possible. Then compare your answers with the correct answers, given at the end of this examination guide.

Directions: Solve the following problems. Do not spend too much time on any one problem.

Notes: (1) In this exam, ln x denotes the natural logarithm of x (that is, logarithm to the base e).
(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which f(x) is a real number.

Be sure to examine the form of the answer choices for each multiple-choice question before you make use of a calculator because, in some cases, the form of the answer may not be readily obtainable on a calculator. For example, the answer choices may involve radicals or numbers such as π and e.

1. If the graph of $y = 2^{-x} - 1$ is reflected in the x-axis, then an equation of the reflection is y =

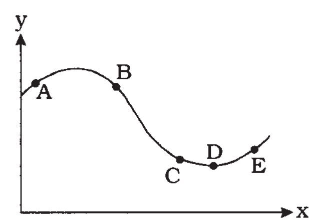
(A) $2^{x} - 1$
(B) $1 - 2^{x}$
(C) $1 - 2^{-x}$
(D) $\log_2 (x + 1)$
(E) $\log_2 (1 - x)$

Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ

2. What is $\lim\limits_{h \to 0} \dfrac{\cos(\frac{\pi}{2} + h) - \cos\frac{\pi}{2}}{h}$?

(A) $-\infty$
(B) -1
(C) 0
(D) 1
(E) $+\infty$

Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ



3. At which of the five points on the graph in the figure above are $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$ both negative?

(A) A
(B) B
(C) C
(D) D
(E) E

Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ

4. If $y = x + \sin(xy)$, then $\dfrac{dy}{dx} =$

(A) $1 + \cos(xy)$

(B) $1 + y\cos(xy)$

(C) $\dfrac{1}{1-\cos(xy)}$

(D) $\dfrac{1}{1 - x\cos(xy)}$

(E) $\dfrac{1 + y\cos(xy)}{1 - x\cos(xy)}$

Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ

5. Which of the following statements about the curve $y = x^4 - 2x^3$ is true?

(A) The curve has no relative extremum.
(B) The curve has one point of inflection and two relative extrema.
(C) The curve has two points of inflection and one relative extremum.
(D) The curve has two points of inflection and two relative extrema.
(E) The curve has two points of inflection and three relative extrema.

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6. If h is the inverse function of f and if $f(x) = \dfrac{1}{x}$, then $h(x) =$

(A) $\ln x$

(B) $-\dfrac{1}{x^2}$

(C) $-\dfrac{1}{x}$

(D) x

(E) $\dfrac{1}{x}$

Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ

7. A smooth curve has the property that for all x the value of its slope at 2x is twice the value of its slope at x. The slope at 0 is

(A) not defined
(B) negative
(C) zero
(D) positive
(E) not determinable from the information given

Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ

8. Let $f(x) = \frac{1}{k}\cos(kx)$. For what value of k does f have period 3 ?

(A) $\frac{2}{3}$
(B) $\frac{2\pi}{3}$
(C) $\frac{3\pi}{2}$
(D) 6
(E) 6π

Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ

9. $\int (x-1)\sqrt{x}\,dx =$

(A) $\frac{3}{2}\sqrt{x} - \frac{1}{\sqrt{x}} + C$
(B) $\frac{2}{3}x^{\frac{3}{2}} + \frac{1}{2}x^{\frac{1}{2}} + C$
(C) $\frac{1}{2}x^2 - x + C$
(D) $\frac{2}{5}x^{\frac{5}{2}} - \frac{2}{3}x^{\frac{3}{2}} + C$
(E) $\frac{1}{2}x^2 + 2x^{\frac{3}{2}} - x + C$

Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ

10. What is $\lim_{x \to \infty} \frac{x^2 - 4}{2 + x - 4x^2}$?

(A) -2

(B) $-\frac{1}{4}$

(C) $\frac{1}{2}$

(D) 1

(E) The limit does not exist.

Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ

11. The area of the region in the first quadrant between the graph of $y = x\sqrt{4 - x^2}$ and the x-axis is

(A) $\frac{2}{3}\sqrt{2}$

(B) $\frac{8}{3}$

(C) $2\sqrt{2}$

(D) $2\sqrt{3}$

(E) $\frac{16}{3}$

Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ

12. For which of the following functions does the property $\frac{d^3y}{dx^3} = \frac{dy}{dx}$ hold?

I. $y = e^x$
II. $y = e^{-x}$
III. $y = \sin x$

(A) I only
(B) II only
(C) III only
(D) I and II
(E) II and III

Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ

13. Let a < c < b and let f be differentiable on [a, b]. Which of the following is NOT necessarily true?

(A) $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$

(B) There exists d in [a, b] such that $f'(d) = \dfrac{f(b)-f(a)}{b-a}$.

(C) $\int_a^b f(x)dx \geqq 0$

(D) $\lim_{x \to c} f(x) = f(c)$

(E) If k is a real number, then $\int_a^b kf(x)dx = k\int_a^b f(x)dx$.

Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ

14. The function $f(x) = \ln(\sin x)$ is defined for all x in which of the following intervals?

(A) $0 < x < \pi$

(B) $0 \leqq x \leqq \pi$

(C) $\dfrac{3\pi}{2} < x < \dfrac{5\pi}{2}$

(D) $\dfrac{3\pi}{2} \leqq x \leqq \dfrac{5\pi}{2}$

(E) $\dfrac{3\pi}{2} < x < 2\pi$

Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ

15. $\int_{-3}^{3} |x+2|\,dx =$

(A) 0
(B) 9
(C) 12
(D) 13
(E) 14

Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ

16. The volume generated by revolving about the x-axis the region enclosed by the graphs of $y = 2x$ and $y = 2x^2$, for $0 \leqq x \leqq 1$, is

(A) $\pi\int_0^1 (2x-2x^2)^2\,dx$

(B) $\pi\int_0^1 (4x^2-4x^4)\,dx$

(C) $2\pi\int_0^1 x(2x-2x^2)\,dx$

(D) $\pi\int_0^2 \left(\sqrt{\frac{y}{2}}-\frac{y}{2}\right)^2 dy$

(E) $\pi\int_0^2 \left(\frac{y}{2}-\frac{y^2}{2}\right) dy$

Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ

17. Let f be defined as follows, where $a \neq 0$.

$$f(x) = \begin{cases} \dfrac{x^2-a^2}{x-a}, & \text{for } x \neq a, \\ 0, & \text{for } x = a. \end{cases}$$

Which of the following are true about f ?

I. $\lim_{x\to a} f(x)$ exists.
II. $f(a)$ exists.
III. $f(x)$ is continuous at $x = a$.

(A) None
(B) I only
(C) II only
(D) I and II only
(E) I, II, and III

Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ

18. Which of the following definite integrals is NOT equal to 0?

(A) $\int_{-\pi}^{\pi} \sin^3 x \, dx$

(B) $\int_{-\pi}^{\pi} x^2 \sin x \, dx$

(C) $\int_{0}^{\pi} \cos x \, dx$

(D) $\int_{-\pi}^{\pi} \cos^3 x \, dx$

(E) $\int_{-\pi}^{\pi} \cos^2 x \, dx$

Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ

19. The acceleration at time t of a particle moving on the x-axis is $4\pi \cos t$. If the velocity is 0 at $t = 0$, what is the average velocity of the particle over the interval $0 \leqq t \leqq \pi$?

(A) 0

(B) $\frac{4}{\pi}$

(C) 4

(D) 8

(E) 8π

Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ

20. $\int_{0}^{1} xe^{-x} dx =$

(A) $\frac{e-2}{e}$

(B) $\frac{2-e}{e}$

(C) $\frac{e+2}{e}$

(D) $\frac{e}{e-2}$

(E) $\frac{e}{2-e}$

Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ

21. Let $f(x) = x^3 + x$. If h is the inverse function of f, then $h'(2) =$

(A) $\frac{1}{13}$

(B) $\frac{1}{4}$

(C) 1

(D) 4

(E) 13

Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ

22. $\int \cos^2 x \sin x \, dx =$

(A) $-\frac{\cos^3 x}{3} + C$

(B) $-\frac{\cos^3 x \sin^2 x}{6} + C$

(C) $\frac{\sin^2 x}{2} + C$

(D) $\frac{\cos^3 x}{3} + C$

(E) $\frac{\cos^3 x \sin^2 x}{6} + C$

Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ

23. If r is positive and increasing, for what value of r is the rate of increase of r^3 twelve times that of r ?

(A) $\sqrt[3]{4}$
(B) 2
(C) $\sqrt[3]{12}$
(D) $2\sqrt{3}$
(E) 6

Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ

24. If f is continuous for all x, which of the following integrals necessarily have the same value?

I. $\int_{a}^{b} f(x)dx$

II. $\int_{o}^{b-a} f(x+a)dx$

III. $\int_{a+c}^{b+c} f(x+c)dx$

(A) I and II only
(B) I and III only
(C) II and III only
(D) I, II, and III
(E) No two necessarily have the same value.

Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ

25. The normal to the curve represented by the equation $y = x^2 + 6x + 4$ at the point (–2, –4) also intersects the curve at x =

(A) -6

(B) $-\frac{9}{2}$

(C) $-\frac{7}{2}$

(D) -3

(E) $-\frac{1}{2}$

Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ

Study Resources

To prepare for the Calculus exam, a candidate is advised to study one or more introductory college level calculus textbooks, which can be found in most college bookstores. When selecting a textbook, check the table of contents against the "Knowledge and Skills Required" section on pages 1-4. In addition, the Barron's book provides helpful test preparation suggestions, and the Schaum Outline provides a condensed version of the important topics usually covered in a college calculus course. Both of these books contain many sample problems; many of those in the Barron's book are taken from old forms of Advanced Placement and CLEP exams.

Answers to Sample Questions

Calculus with Elementary Functions

1. C
2. B
3. B
4. E
5. C
6. E
7. C
8. B
9. D
10. B
11. B
12. D
13. C
14. A
15. D
16. B
17. D
18. E
19. D
20. A
21. B
22. A
23. B
24. A
25. B